RHEOLOGICAL BEHAVIOR OF DILUTE SUSPENSIONS OF RIGID AXISYMMETRIC PARTICLES

E. Yu. Taran and Yu. V. Pridatchenko

UDC 532.135:541.182

The rheological equations of state of dilute suspensions of rigid axisymmetric particles lacking a central symmetry in a Newtonian dispersion medium are derived. A free-through flow asymmetric, triaxial dumbbell is used as the hydrodynamic model of dispersed particles. The article is concerned with the effect of the asymmetry of dispersed particles and their percentage elongation on their angular velocity and the shape and dimensions of their migration trajectories relative to the dispersion medium and on the effective suspension viscosity in a simple shear flow.

Many basic processes in a living cell (fission, transfer of characteristics, and mutability) occur at the molecular level. Thus, the problems of the structure and function of molecules of biologically active polymers – proteins and nucleic acids – have acquired fundamental importance in modern natural science.

Adequate hydrodynamic models of macromolecules are now very important in investigating experimentally the structure and characteristics of macromolecules in solutions by studying the viscous flow of macromolecule solutions and their translational and rotational friction during diffusion and sedimentation and phenomena of double refraction in a laminar flow.

A characteristic of the most important biological polymers – proteins and nucleic acids – is the constancy of rigid conformations of their molecular chains. Thus, the macromolecules of native proteins, for instance, trypsin, egg albumen, and human serous protein, in both globular and spiralized states, behave as rigid particles, whose shape can be simulated by the hydrodynamic model of an ellipsoid of rotation [1]. A spheric model is used [1] in investigating the structure and form of the bean mosaic virus – a particle which has a permolecular structure. These hydrodynamic models, which have been introduced in [2-4], can be used only in those cases where the macromolecules or particles possessing a permolecular structure are characterized either by central spherical asymmetry or axial symmetry with a symmetry center.

In many cases, the actual shape of protein macromolecules characterized by a rigid conformation is far from being spherical or ellipsoidal. For instance, the molecules of myoglobin – muscle protein – are asymmetric particles [1]. The lack of an adequate hydrodynamic model of asymmetric macromolecules makes it impossible to use the above mentioned experimental methods for investigating their shape and structure. We propose here a hydrodynamic model of rigid axisymmetric macromolecules or particles with a permolecular structure lacking a central symmetry.

The theorem that any axisymmetric dispersed particle lacking a central symmetry (henceforth, asymmetric particle, for brevity) must rotate like some equivalent ellipsoid of rotation in a simple shear flow of a dispersion medium has been proved in [5]. In spite of this, neither an ellipsoid of rotation nor a symmetric triaxial dumbbell, which is hydrodynamically equivalent to the former [6], can be used as the hydrodynamic model of asymmetric dispersed particles. This is due to the fact that, as has been shown in [7, 8], such particles must experience, besides rotational motion, also translational migration relative to the dispersion medium in a simple shear flow. For the hydrodynamic model of dispersed particles, an ovoid – an egg-shaped body – was used as a simple asymmetric non-through flow model of such particles. Due to the complexity of the ovoid surface, the problem of hydrodynamic interaction between this model and the liquid ambient was solved [7, 8]. Only for Bretherton's ovoid was the cause of the smallness of its migration trajectory. The extent of the

T. G. Shevchenko Kiev State University. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 62, No. 1, pp. 57-65, January, 1992. Original article submitted January 8, 1991.

1062-0125/92/6201-0044$12.50

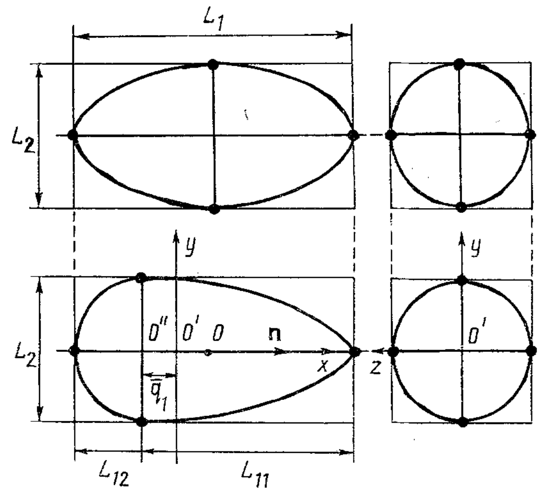

Fig. 1. Hydrodynamic models of symmetric and asymmetric dispersed particles: non-through flow models — ellipsoid of rotation and ovoid; free-through flow models — symmetric and asymmetric triaxial dumbbells.

orbit in the direction of the velocity vector in a simple shear flow amounted to only 5% of the radius of the sphere, the deformation of which produces the ovoid [8].

For investigating the motion of particles with an arbitrary or even considerable, asymmetry, the migration orbits of which can be measured experimentally, it is suggested to use, instead of an ovoid, a free-through flow model — an asymmetric, triaxial, dumbbell (henceforth, asymmetric dumbbell, for brevity) (Fig. 1). By analogy with [6], where a triaxial dumbbell was used to stimulate axisymmetric dispersed particles possessing a central symmetry, the principal axis L_1 of the dumbbell is equal to the length of the symmetry axis of the ovoid, while the other two, L_2 and L_3, are equal to each other and to the diameter of its largest transverse cross section (Fig. 1). The L_2 and L_3 axes are perpendicular to each other and to the L_1 axis, which they subdivide into two unequal parts, L_{11} and L_{12} ($L_{11} > L_{12}$). The value of $q = (L_{11}-L_{12})/L_1$ is used as the measure of asymmetry of the triaxial dumbbell.

As in [6], it is assumed that point centers of hydrodynamic interaction between the model and the liquid ambient are located at the ends of the axes. This means that, if the dispersion medium flows around the end of the dumbbell axis at the velocity U_i, it is acted upon by the force ξU_i on the part of the liquid. The dumbbell axes do not offer hydrodynamic resistance.

In deriving the equations of motion of an asymmetric dumbbell, it is assumed that the asymmetric particle represented by the above model has, on the one hand, such dimensions that the dispersion medium interacts with it as with a hydrodynamic body, while, on the other, the dispersed particle must be sufficiently small for the velocity of the dispersion medium in its neighborhood to be a homogeneous function of the coordinates, i.e., $v_i=\omega_{ik}r_k+d_{ik}r_k$. The radius vector r_k determines the position of a point in the vicinity of the particle in the (x', y', z') coordinate system bound to it, whose origin is at the reaction center O' of the asymmetric dumbbell (Fig. 1). The reaction center of the asymmetric dumbbell lies on the L_1 axis, which is due to the axial symmetry of the model under consideration. According to [9], its position at the axis is determined by setting equal to zero the moment of hydrodynamic forces acting on the asymmetric dumbbell, which is hinged at its reaction center and is located in the plane-parallel flow of the dispersion medium. It is found that the reaction center O' lies at the distance $\bar{q}_1=(L_{11}-L_{12})/4$ from the point of intersection between the dumbbell axes. The vector of hydrodynamic forces F_i acting on the asymmetric dumbbell in an arbitrary velocity field v_i of the dispersion medium is defined as the sum of the forces acting at the ends of its axes:

$$F_i=\xi L_1\left\{\frac{q}{2}[\dot{n}_i-(d_{ij}+\omega_{ij})n_j]-6v_{0i}\right\}. \tag{1}$$

TABLE 1. Relative Transverse ($\bar{l}_x = l_x/L_1$) and Longitudinal ($\bar{l}_y = l_y/L_1$) Dimensions of Migration Trajectories of Asymmetric Triaxial Dumbbells in a Simple Shear Flow

	q	0,1	0,2	0,3	0,5	0,7	0,9	1,0
$\bar{l}_x$		0,0167	0,0333	0,0500	0,0833	0,1167	0,1500	0,1667
$\bar{l}_y$	$p=1$	0,0166	0,0330	0,0491	0,0795	0,1078	0,1345	0,1477
	$p=2$	0,0150	0,0302	0,0455	0,0770	0,1106	0,1470	0,1664
	$p=3$	0,0180	0,0362	0,0547	0,0936	0,1358	0,1821	0,2069
	$p=4$	0,0211	0,0425	0,0643	0,1101	0,1598	0,2141	0,2432
	$p=5$	0,0239	0,0481	0,0728	0,1246	0,1807	0,2417	0,2742
	$p=6$	0,0264	0,0531	0,0804	0,1374	0,1988	0,2655	0,3009
	$p=7$	0,0286	0,0576	0,0871	0,1486	0,2148	0,2863	0,3241
	$p=8$	0,0306	0,0615	0,0930	0,1587	0,2290	0,3047	0,3447
	$p=9$	0,0324	0,0651	0,0984	0,1677	0,2417	0,3213	0,3632
	$p=10$	0,0340	0,0684	0,1033	0,1390	0,1759	0,2533	0,3360
	$p=12$	0,0369	0,0741	0,1119	0,1902	0,2735	0,3624	0,4090
	$p=15$	0,0404	0,0812	0,1225	0,2081	0,2986	0,3948	0,4451
	$p=18$	0,0434	0,0871	0,1314	0,2229	0,3193	0,4216	0,4749
	$p=20$	0,0451	0,0905	0,1365	0,2315	0,3314	0,4371	0,4921

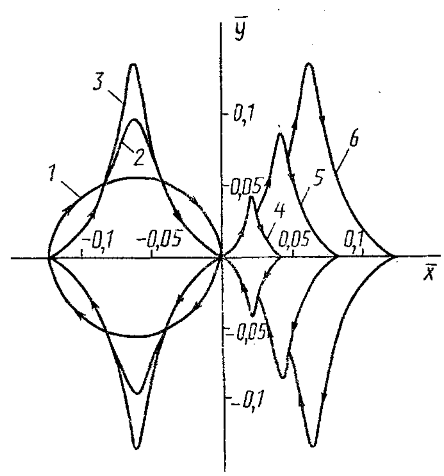

Fig. 2. Trajectories of translational migration of asymmetric dumbbells in a simple shear flow of a dispersion medium ($\bar{x}=x/L_1$, $\bar{y}=y/L_1$) : 1), 2), and 3) at q = 0.75; p = 1, 5, and 10, respectively; 4, 5), and 6) at p = 10; q = 0.25, 0.5, and 0.75, respectively.

It is assumed that, besides the hydrodynamic forces, also external forces – electric and Brownian forces – can act on a suspended asymmetric dumbbell; the moment M_i about the particle's reaction center is determined by the relationship

$$M_i = -\frac{1}{2}\xi\varepsilon_{ijk}\left\{2\left(L_{11}^2 + L_{12}^2 + \frac{3}{8}q^2\right)n_j\left[(d_{kl}+\omega_{kl})n_l - \dot{n}_k\right] + \right.$$
$$\left. + L_2^2 m_j\left[(d_{kl}+\omega_{kl})m_l - \dot{m}_k\right] + L_2^2 k_j\left[(d_{kl}+\omega_{kl})k_l - \dot{k}_k\right] - qL_1 n_j v_{0k}\right\} + M_i^*. \qquad (2)$$

If the moment of inertia of a dispersed particle is neglected, the equations of motion of the asymmetric dumbbell under the action of hydrodynamic and external forces assume the form $F_i + F_i^* = 0$; $M_i = 0$ and, with an allowance for (1), (2), and the condition $n_i M_i^* = 0$ are written as follows:

$$\dot{n}_i = \omega_{ij} n_j + \tilde{\lambda}(d_{ik} n_k - d_{km} n_k n_m n_i) - \frac{1}{\tilde{\gamma}} \varepsilon_{ikm} n_k M_m^* - \frac{\alpha}{\tilde{\lambda}\tilde{\gamma}} (F_i^* - F_k^* n_k n_i), \tag{3}$$

$$v_{0i} = -\frac{2\alpha}{p_e^2 - 1} d_{ik} n_k + \alpha d_{km} n_k n_m n_i + \frac{\alpha}{\tilde{\lambda}\tilde{\gamma}} \varepsilon_{ikm} n_k M_m^* + \left(\frac{\alpha^2}{\tilde{\lambda}^2 \tilde{\gamma}} + \frac{1}{6\xi}\right) F_i^* - \frac{\alpha^2}{\tilde{\lambda}^2 \tilde{\gamma}} F_k^* n_k n_i, \tag{4}$$

where

$$\tilde{\lambda} = \frac{p_e^2 - 1}{p_e^2 + 1}, \; p_e^2 = p^2\left(1 + \frac{2}{3} q^2\right), \; p = \frac{L_1}{L_2};$$

$$\tilde{\gamma} = \frac{1}{2} \xi L_2^2 (p_e^2 + 1); \; \alpha = -\frac{1}{12} L_1 q \tilde{\lambda}.$$

For $F_i^* = 0$ and $M_i^* = 0$, Eq. (3) characterizing the rotation of an asymmetric dumbbell relative to the reaction center O' coincides with the equation of rotational motion of the equivalent ellipsoid of rotation whose axis ratio is

$$p_e = p\left(1 + \frac{2}{3} q^2\right)^{\frac{1}{2}} \tag{5}$$

in an arbitrary gradient flow of a Newtonian liquid [10]. This result not only agrees with the inferences of the theorem concerning the rotation of an arbitrary asymmetric particle in a simple shear flow of a dispersion medium, which has been proved in [5], but also generalizes them to encompass an arbitrary flow in the case of rotation of an asymmetric dumbbell.

According to (5), $p_e > p$ for $0 < q \leq 1$. The difference between the actual axis ratio $p = L_1/L_2$ of an asymmetric dumbbell and the axis ratio p_e of an equivalent ellipsoid of rotation can amount to more than 28% for large values of $q \leq 1$. This is what primarily hinders the use of the equivalent ellipsoid as a hydrodynamic model of asymmetric macromolecules in determining their dimensions, for instance, in experiments on double refraction in the laminar flow of a dilute solution of such macromolecules.

According to (3) and (5), the rotation of a symmetric, triaxial dumbbell for $q = 0$ coincides with the rotational motion of an ellipsoid of rotation that is equiaxial with the former, i.e., an ellipsoid of rotation with the axis ratio $Pp = L_1/L_2$ (Fig. 1). According to (4), there is no dumbbell migration relative to the dispersion medium: $v_{0i} \equiv 0$.

It follows from (3) and (4) that the orientation vector n_i coinciding with the principal axis L_1 of the particle is sufficient for describing the motion of asymmetric particles in the gradient flow of a dispersion medium. Therefore, the structural – phenomenological approach, utilized, for instance, in [6, 10], is used for deriving the rheological equation of state for the stress in a dilute suspension of asymmetric particles.

According to [6], the stress tensor in such a suspension should be sought in the form of the phenomenological relationship

$$\begin{aligned} T_{ij} = \tau_{ij} + N_0 (a_1 \langle n_i n_j \rangle + a_2 d_{km} \langle n_k n_m n_i n_j \rangle + \\ + a_3 d_{ij} + a_4 d_{ik} \langle n_k n_j \rangle + a_5 d_{jk} \langle n_k n_i \rangle + \\ + a_6 \langle n_i (\dot{n}_j - \omega_{jk} n_k) \rangle + a_7 \langle n_j (\dot{n}_i - \omega_{ik} n_k) \rangle \cdot \end{aligned} \tag{6}$$

The averaging in (6) is realized by means of the distribution function of the axes of dispersed particles with respect to angular positions, which constitutes the solution of the equation

$$\frac{\partial F}{\partial t} + \frac{\partial (F \dot{n}_i)}{\partial n_i} = 0. \tag{7}$$

Comparing, as in [6], the expressions for the energy dissipation rate per unit volume of suspension, calculated on the basis of the structural and the phenomenological theories, we obtain

$$a_1 = a_2 = 0,\ a_3 = 2\xi L_2^2,\ a_6 + a_7 = -(a_4 + a_5),$$
$$a_4 + a_5 = \xi/6\,(5L_{11}^2 + 5L_{12}^2 + 2L_{11}L_{12} - 12L_2^2). \tag{8}$$

By substituting relationship (3) in (6), assuming that the external moment M^*_i is due only to Brownian forces, we obtain the following with an allowance for (8):

$$T_{ij} = \tau_{ij} + 2\mu_0 d_{ij} + \mu_1 \langle n_i n_j \rangle +$$
$$+ \mu_2 d_{km} \langle n_k n_m n_i n_j \rangle + 2\mu_3 (d_{ik} \langle n_k n_j \rangle + d_{jk} \langle n_k n_i \rangle). \tag{9}$$

Here,

$$\mu_0 = \frac{1}{4} N_0 \omega,\ \mu_1 = 3D_r N_0 \tilde{\gamma} \tilde{\lambda},\ \mu_2 = N_0 \tilde{\lambda}^2 \tilde{\gamma},$$
$$\mu_3 = \frac{1}{4} N_0 \tilde{\lambda} \omega;\ \omega = \xi L_2^2.$$

In order to determine by means of (3) and (4) the effect of asymmetry of a particle on its angular velocity and translational migration relative to the dispersion medium for $F_i^* = 0$ and $M_i^* = 0$, we consider the motion of the asymmetric dumbbell in a simple shear flow:

$$v_x = 0,\ v_y = Kx,\ v_z = 0;\ K - \text{const}. \tag{10}$$

We use the (x, y, z) laboratory coordinate system and consider the case where the vector n_i characterizing the orientation of the asymmetric dumbbell in this system has the coordinates $n_x = \cos\varphi$, $n_y = \sin\varphi$ and $n_z = 0$. Equations (3) and (4) then assume the following form:

$$\dot{\varphi}_t = \frac{K}{2}(1 + \tilde{\lambda}\cos 2\varphi), \tag{11}$$

$$v_{0x} \equiv \dot{x}_t = \frac{\alpha}{p_e^2 - 1} K \sin^3\varphi\,(1 + p_e^2 \operatorname{ctg}^2\varphi), \tag{12}$$

$$v_{0y} \equiv \dot{y}_t = \frac{\alpha}{p_e^2 - 1} K \cos^3\varphi\,(1 + p_e^2 \operatorname{tg}^2\varphi). \tag{13}$$

According to (11), the asymmetry of the triaxial dumbbell causes its maximum angular velocity to increase at $\varphi = 0$ and 180°C and its minimum angular velocity to diminish at φ = 90 and 270°C in comparison with the angular velocity of an equiaxial symmetric dumbbell or an ellipsoid of rotation.

The relationship between the angular velocity of the asymmetric dumbbell and its angular position, in turn, causes the kinematic orientation of the dumbbell's principal axis to assume the direction of the velocity vector. The nonuniform distribution of the principal axes of the dumbbells with respect to the angle φ is characterized, as in [11], by the distribution function

$$P(\varphi) = \frac{p_e}{2\pi(p_e^2\cos^2\varphi + \sin^2\varphi)}.$$

The asymmetry of the triaxial dumbbell makes the maximum of the P(φ) function larger at φ = 90 and 270°C, i.e., it reinforces the predominant orientation of the principal axes of triaxial dumbbells in the direction of the flow in comparison with equiaxial, symmetric dumbbells or ellipsoids of rotation.

The solution of Eq. (11), which determines the angular velocity of an asymmetric dumbbell relative to the reaction center for the initial condition φ = 0 at t = 0, makes it possible to determine the rotation period of the asymmetric dumbbell:

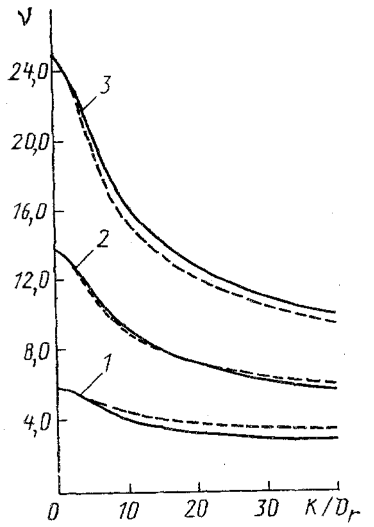

Fig. 3

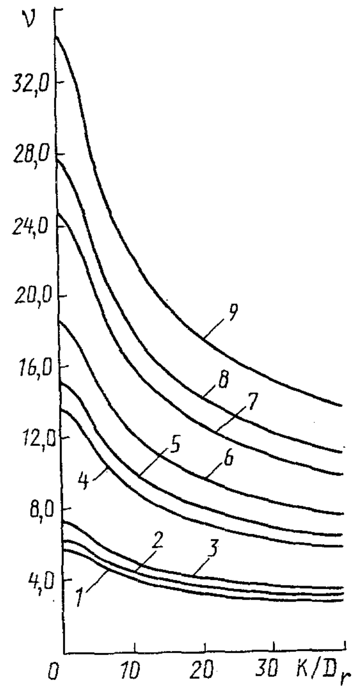

Fig. 4

Fig. 3. Increment of the viscosity of a dilute suspension of particles simulated by triaxial dumbbells at q = 0 (solid curves) and of ellipsoidal particles (dashed curves) as a function of the dimensionless shear velocity K/D_r. 1), 2), and 3) at p = 5, 10, and 15, respectively.

Fig. 4. Increment of the viscosity of a dilute suspension of asymmetric dumbbells as a function of the dimensionless shear velocity K/D_r. 1), 2), and 3) at p = 5; q = 0, 0.5, and 0.9, respectively; 4), 5), and 6) at p = 10; q = 0, 0.5, and 0.9, respectively; 7), 8) and 9) at p = 15; q = 0, 0.5, and 0.9, respectively.

$$T = \frac{2\pi}{K}\left(p_e + \frac{1}{p_e}\right). \tag{14}$$

Since $p_e > p$ for $q \neq 0$, the asymmetry of the triaxial dumbbell, according to (14), causes its rotation period to increase in comparison with that of an equiaxial symmetric dumbbell or an ellipsoid of rotation.

Solving Eqs. (12) and (13) for the initial conditions x = y = 0 at $\varphi=\varphi_0$ by means of the substitution $\dot{x}_t=\dot{x}_\varphi\dot{\varphi}_t,\ \dot{y}_t=\dot{y}_\varphi\dot{\varphi}_t$ and using Eq. (11) for $\dot{\varphi}_t$, we obtain the parametric equations of the trajectories of translational migration of an asymmetric dumbbell in the simple shear flow (10) of the dispersion medium:

$$x = \frac{\alpha}{\tilde{\lambda}}(\cos\varphi_0 - \cos\varphi), \tag{15}$$

$$y = \frac{\alpha}{\tilde{\lambda}}\left(\sin\varphi_0 - \sin\varphi + \frac{1}{2p_e}\sqrt{\frac{p_e^2+1}{\tilde{\lambda}}}\ln\frac{(p_e+\sqrt{p_e^2-1}\,\sin\varphi)(p_e-\sqrt{p_e^2-1}\,\sin\varphi_0)}{(p_e+\sqrt{p_e^2-1}\,\sin\varphi_0)(p_e-\sqrt{p_e^2-1}\,\sin\varphi)}\right), \tag{16}$$

where $0 \leqslant \varphi \leqslant 2\pi$. According to (15) and (16), the migration trajectories in the case $F_i^* = 0$, $M_i^* = 0$, which is contemplated here, have two mutually perpendicular symmetry axes, parallel to the Ox and Oy axes, while they are independent of the shear of flow (10). The trajectory shape is not affected by the initial angular position φ_0 of the asymmetric dumbbell either. Changes in φ_0 only cause variation in the trajectory position relative to the coordinate axes. The shape and dimensions of the trajectory are fully determined by the parameters p, q, and L_1, which characterize an asymmetric dumbbell (Fig. 2). Table 1 provides the relative dimensions of the migration trajectories of asymmetric dumbbells, obtained by means of expressions (15) and (16).

We did not compare the results obtained with experimental data or solutions of problems of motion of actual asymmetric particles (macromolecules), as there were not available. Comparison with the approximate solution of the problem involving the motion of Bretherton's ovoid (q = 0.1) in a simple shear flow [8] indicates that there is qualitative agreement between the rotational and the migratory motions of an ovoid and those of an asymmetric, triaxial dumbbell (p = 1; q = 0.1).

If the motion of actual dispersed asymmetric particles can be observed visually, the parameters p, q, and L_1 necessary for simulating them can be determined by using relationships (14)-(16) and experimental data on the longitudinal and transverse dimensions of migration trajectories and the rotation period of these particles in flow (10). If visualization of the motion of dispersed asymmetric particles (macromolecules) is impossible, then, the values of p, q, and L_1 should be determined by means of experiments on the viscous flow of dilute suspensions of such particles (macromolecules) in a laminar flow [1], introducing in the experimental methodology suitable changes which would allow us to take into account the asymmetry of dispersed particles.

In order to determine by means of (7) and (9) the effect of the asymmetry of dispersed particles on the rheological behavior of a dilute suspension, we consider the shear flow (10). For the effective viscosity of the suspension μ_{eff}, we obtain the relationship

$$\mu_{eff} = \frac{T_{yx}}{K} = \mu + \mu_0 + \frac{\mu_1}{2K} \langle \sin 2\varphi \sin^2\theta \rangle + \frac{1}{4}\mu_2 \langle \sin^2 2\varphi \sin^4\theta \rangle + \mu_3 \langle \sin^2\theta \rangle . \tag{17}$$

For the averaging in (17), we use the distribution function obtained in [12] in the form of a spherical harmonics series as the solution of Eq. (7) for the flow (10).

In the absence of experimental or theoretical data on suspensions of asymmetric particles, we compared the viscosity increments $\nu = (\mu_{eff} - \mu)/\mu V$ for suspensions of ellipsoidal particles and particles simulated by symmetric (q = 0) triaxial dumbbells with the same axis ratio. In connection with the fact that a free-through flow particle does not possess volume, the latter is ascribed to it on the basis of agreement between the effective viscosities of the suspensions in question at the zero shear velocity.

This comparison indicates that the triaxial dumbbell model is a good approximation of the corresponding non-through flow particles (Fig. 3).

It was found that greater asymmetry of dispersed particles produces a higher suspension viscosity, especially at low shear velocities and/or large particle elongations (Fig. 4). The effect of elongation of asymmetric particles on the effective viscosity of the suspension is similar to that in the case of a suspension of ellipsoidal particles — greater elongation leads to a higher suspension viscosity.

Conclusions. An asymmetric, triaxial dumbbell in a gradient flow displays the behavior characteristic of axisymmetric dispersed particles without a central symmetry. This suggests that we can use it as the hydrodynamic model of such particles and utilize its equations of motion (3) and (4) in writing the rheological equation of state of dilute suspensions (9). The asymmetry of dispersed particles leads to an increase in the effective viscosity of such suspensions in a simple shear flow.

NOTATION

L_1, L_2, and L_3, axes of an asymmetric triaxial dumbbell; q, measure of asymmetry of a triaxial dumbbell; U_i flow velocity of the dispersion medium around the end of the dumbbell; ξ, coefficient of translational friction of the dumbbell end in the dispersion medium; v_i, velocity of the dispersion medium; ω_{ik}, tensor of the velocity vortex; d_{ik}, strain rate tensor; r_k, radius vector of a point in the neighborhood of the dispersed particle; (x', y', z') coordinate system bound to the dispersed particle, the origin of which is at the reaction center O'; F_i, vector of the hydrodynamic forces acting on an asymmetric dumbbell; n_i, m_i, k_i, basis vectors of the (x', y', z'); coordinate system; $\dot{n}_i$, $\dot{m}_i$, $\dot{k}_i$, time derivatives; v_{oi}, migration velocity of the reaction center of an asymmetric dumbbell relative to the dispersion medium; M_i and M_i^*, moment of forces and moment of external forces F_i^* acting on the asymmetric dumbbell, respectively; ε_{ijk} , Levi-Civita tensor; p, axis ratio of an asymmetric particle characterizing its elongation; p_e, axis ratio of the ellipsoid of rotation equivalent to the asymmetric dumbbell; T_{ij}, stress tensor in a dilute suspension of asymmetric particles; τ_{ij}, stress tensor in the dispersion medium in the absence of dispersed particles; N_0, number of dispersed particles per unit volume of suspension; α_i (i = 1, ..., 7), and μ_i (i = 0, 1, 2, 3), rheological constants; <>, symbol of averaging by means of the distribution function F; ω, coefficient of rotary friction of an asymmetric particle relative to the L_1 axis; D_r, coefficient of rotary Brownian diffusion of asymmetric dispersed particles; v_x, v_y, and v_z, coordinates of the v_i vector; K, shear velocity in a simple shear flow; (x, y, z), laboratory coordinate system; φ, angle between the Ox axis and the n_i vector; $\dot{\varphi}_t$, $\dot{x}_t$, $\dot{y}_t$ derivatives with respect to time; v_{ox} and v_{oy}, coordinates of the v_{oi} vector in the (x, y, z), coordinate system; φ_0 , initial angular position of the asymmetric dumbbell; $P(\varphi)$, distribution function of dumbbell axes with respect to the angle φ without an allowance for the rotary Brownian motion of the dumbbells; T, rotation period of an asymmetric dumbbell; μ_{eff}, effective viscosity of the dilute suspension of asymmetric particles in a simple shear flow; μ, viscosity of the dispersion medium; φ and θ, azimuthal and latitude angles in a spherical coordinate system that determine the angular position of a dispersed particle in space.

LITERATURE CITED

1. V. N. Tsvetkov, V. E. Eskin, and S. Ya. Frenkel', Structure of Macromolecules in Solutions [in Russian], Moscow (1964).
2. A. Einstein, Ann. Physik, 19, 289-306 (1906).
3. A. Einstein, Ann. Physik, 34, 591-592 (1911).
4. G. B. Jeffery, Proc. R. Soc., A102, No. 715, 161-179 (1922).
5. F. P. Bretherton, J. Fluid Mech., 14, No. 2, 234-304 (1962).
6. Yu. V. Pridatchenko and E. yu. Taran, Dokl. Akad. Nauk USSR, Ser. A, No. 10, 59-62 (1988).
7. H. Brenner, Chem. Eng. Sci., 19, No. 9, 631-651 (1964).
8. C. E. Chaffey, M. Takano, and S. G. Mason, Can. J. Physics, 43, No. 7, 1269-1287 (1965).
9. J. Happel and H. Brenner, Hydrodynamics at Small Reynolds Numbers [Russian translation], Moscow (1976).
10. Yu. I. Shmakov and E. Yu. Taran, Inzh.-fiz. Zh., 18, No. 6, 1019-1024 (1970).
11. S. G. Mason and R. St. J. Manley, Proc. R. Soc., A238, No. 1212, 117-131 (1956).
12. A. Peterlin, Z. Phys., 111, 232-263 (1938).